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The Macroeconomic Effects of Infrequent Information with Adjustment Costs

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Abstract

We extend the macroeconomic literature on Ss type rules by introducing infrequent information in a kinked adjustment cost model. We first show that optimal individual decision rules are both state-and-time dependent. We then develop an aggregation framework to study the macroeconomic implications of such optimal individual decision rules. In our model, a vast number of agents act together, and more so when uncertainty is large. The average effect of an aggregate shock is inversely related to its size and to aggregate uncertainty. These results are in contrast with those obtained with full information adjustment cost models.

1 Introduction

In the last decade, the macroeconomic literature paid considerable attention to the potential aggregate effects of intermittent large adjustments in microeconomic decision variables[1]. Examples of decision variables modeled in that way are prices, investment, consumption of durables and employment[2]. A distinctive feature of this literature is that explicit aggregation of individual rules is undertaken, resulting in rich dynamic patterns for aggregate variables which are in sharp contrast with the inertial microeconomic behavior. In these adjustment cost models, economic agents always observe the frictionless optimal level of the control variable and infrequent adjustments may be justified by optimal behavior in the presence of kinked adjustment costs (Bertola and Caballero, 1990)[3][4].

We develop a simple model which introduces imperfect information in a kinked adjustment cost model by assuming that agents do not observe continuously the frictionless optimal level of the control variable. They receive infrequently a flow of information which is considered exogenous to the

[1] We are referring to models of state-dependent rules. In fact, ten years before the combination of time-dependent pricing rules became an essencial ingredient for the Keynesian reaction to the Rational Expectations revolution. However, the focus was in only one microeconomic decision variable: price. Examples of macroeconomic models built on time-dependent pricing rules are Fischer (1977), Taylor (1979), and more recently, Ball (1994), Ireland (1997), and Bonomo and Carvalho (1999).

[2] For prices, see Caplin and Spulber (1987), Caplin and Leahy (1991), Caballero and Engel (1992 and 1993), Tsiddon (1993) and Almeida and Bonomo (1999); for investment, Caballero and Engel (1999); for inventories, Caplin (1985); for consumption of durables, Caballero (1993) and for employment, Caballero, Engel and Haltiwanger (1994).

[3] When the adjustment cost function has a kink at the point of no-adjustment, it is best for the agent not to adjust for small changes of the frictionless optimal level. Adjustment is triggered when the discrepancy between the control variable and its optimal level becomes large enough.

[4] Some recent papers extended the state-dependent rules framework to a general equilibrium setting (e.g. Caplin and Leahy 1997, and Danzinger 1999, for pricing rules, and Veracierto 1997, for irreversible investment). Caballero and Engel (1999) model investment decisions using a generalized Ss approach, where adjustment hazards result from stochastic adjustment costs.

agent's decision. Examples of such exogenous intermittent flows are pervasive: macroeconomic statistics such as inflation, level of employment or GNP are published periodically, dividends of firms are announced only at certain dates, information arrives in asset markets after regular closings on weekdays and holidays. In all these cases, agents do not observe continuously the variable of interest. Such an intermittent information arrival has the interesting implication that a large number of agents receive the same information at the same time, creating the conditions for a potential mass reaction[5]. Indeed, increased volatility of financial markets around dividend announcements and macroeconomic data releases have been documented in numerous articles[6].

Our model has the distinct characteristic that a vast number of agents tend to act together, and more so when uncertainty is large[7]. We show that lump-sum adjustment costs interact with infrequent information to generate effects of aggregate shocks on macroeconomic variables that differ substantially from those obtained with continuous information adjustment cost models. First, the relative effect of cumulative aggregate shocks decreases sharply with the size of the shock. Second, the relative average effect of these shocks decreases when aggregate uncertainty increases. Other results are more similar in both models. When idiosyncratic uncertainty increases, the average effect decreases, while an increase in the adjustment cost raises

[5] Imperfect information could also result from the optimal decision of the agent to gather information infrequently in the presence of information collection costs. This endogenous infrequent gathering would not necessarily coordinate agents' reactions and will have different macroeconomic implications. We focus our attention only on the exogenous arrival of macroeconomic information for all agents at the same time.

[6] See Cornell (1978), for dividend announcements and Harvey and Huang (1992), Ederington and Lee (1993), for macroeconomic data releases.

[7] Other authors have explained these mass reactions by different information extraction mechanisms. Banerjee (1992) proposes a model where individuals tend to act simultaneously, even when their private information would not bring by itself such coordination. Caplin and Leahy (1994) provide a rationale for market collapses or crashes based on a discontinuous evolution of public information, which results from difficulties in aggregating private information.

the average effect.

In order to perform the comparisons above, we first solve the microeconomic problem of finding the optimal policy in the presence of both lump-sum adjustment costs and infrequent information about the value of the frictionless optimal level of the control variable. To make the conditions which determine the optimal policy as simple as possible while keeping the main insights of the model, we assume that the stochastic process of the frictionless optimal value of the control variable has no drift. We find that the optimal rule is for agents to adjust or not depending on the state at times of information arrivals.[8] Therefore, it is both state and time dependent. Such a rule was conjectured by Blanchard and Fischer (1989, p. 413) and Caballero (1989) as the rule that could result from a combination of infrequent information about the state variable and adjustment costs. The difference with our model is that they justify the infrequent flow of information by the existence of costs of collecting information.

The optimal rule is characterized by a single parameter s, which determines the inaction range $(-s; s)$ for the discrepancy between the frictionless optimal value of the control variable and its actual value, at times of information arrival. We show that the inaction barriers are much tighter than in the continuous information model. When the adjustment cost is sufficiently low, the barriers are quite insensitive to the uncertainty governing the stochastic process assumed for the optimal level of the control variable. On the other hand, an increase in the adjustment cost brings about a relatively larger increase in the barriers when information is infrequent than when it is continuous.

Ball and Mankiw (1994) also explore the consequences of a price rule that is both time and state dependent. The agents adjust without paying a

[8] The presence of a large drift will make it optimal for agents to adjust between information collections. The conditions determining the optimal policy in the presence of a drift are quite complex.

menu cost at even periods. Adjustments at odd periods will be made only if the benefit of doing so is greater than the menu cost. The frictionless optimal price is always known. They focus mainly on the effect of the drift in the frictionless optimal price process on output dynamics. In our model, we assume the drift to be zero and adjustment costs are always present. Our main goal is to illustrate the interaction between adjustment costs and infrequent information.

The rest of the paper is organized as follows. In Section 2, we derive the optimal rule in the presence of both lump-sum adjustment costs and infrequent observation of the optimal level of the control variable. Section 3 evaluates the aggregate effect of macroeconomic shocks through the optimal adjustment of microeconomic units. Concluding remarks are presented in Section 4.

2 The Optimal Rule

In this section, we set up the optimization problem of agents confronted with infrequent information and adjustment costs and derive the optimal decision rule. We also investigate the implications of this rule for various configurations of adjustment costs and uncertainty in relation with the continuous information case.

2.1 Assumptions and formulation of the optimization problem

An agent faces the problem of setting optimally the level of a control variable x; be it price, employment or investment for a firm, or consumption of some durable good for a household, in the presence of two types of costs: a lump-sum adjustment cost, k, when resetting, and an instantaneous flow cost when its control variable drifts away from a frictionless optimal level, x^*:

For simplicity, we will assume a quadratic form $(x - x^*)^2$ for the latter cost[9]. Time is discounted by a constant instantaneous rate ρ. We depart from the previous literature by assuming that information about the optimal level x^* arrives at discrete time intervals[10]. Although the agent does not observe x^* between two successive information arrivals, he can form probabilistic assessments about the value of x^* given his information, which consists of the past observations of x^* at the discrete information times. We assume, again for simplicity, that x^* is a driftless Brownian motion with diffusion parameter σ; that is:

$$dx_t^* = \sigma dW_t; \tag{1}$$

where W is a Wiener process.[11]

The distribution of x_t^*; conditional on past observations of x^* at the discrete information times, depends only on the last observation x_u^*, where u is the time of the last information arrival. The distribution of x_t^* conditioned on the knowledge of x_u^*, for $u < t$, is normal with mean zero and variance $\sigma^2(t - u)$.

Given initial values for the control variable and the frictionless optimal level, the agent minimizes the expected present value of both the adjustment cost and the flow cost of deviating from the frictionless optimal level of the control variable. The expected value, at the time of the last information arrival u, of the flow cost at time t is $E_u(x - x_t^*)^2$ and can be decomposed as follows:

$$E_u(x - x_t^*)^2 = (x - E_u x_t^*)^2 + E_u(x_t^* - E_u x_t^*)^2$$

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[9] Quadratic flow costs could be justified as a second-order approximation to the loss in profit or utility caused by a non-optimal level of the control variable.

[10] An excellent exposition of optimal control problems under adjustment costs when the frictionless optimal value of the control variable is always known is found in Dixit (1993).

[11] The assumption of an exogenous process for x^* might appear unrealistic in several settings. However, modeling x^* with an endogenous component remains specific to the particular setting considered. We believe that the main insights derived from this simple model will remain unchanged even if the x^* process is partly endogenous.

The second term represents the irreducible cost of not being informed about the optimal value of the frictionless optimal value x^*_t. If there were no adjustment costs, the agent will minimize the expected quadratic flow costs by setting x_t equal to $E_u x^*_t$. Since x^* is driftless, it is a martingale, and $E_u x^*_t = x^*_u$, the value of the optimal variable when the last information arrived. Therefore, even in the absence of adjustment costs, there will be no adjustment between information arrivals[12].

From the structure of the problem and from the Markovian nature of the stochastic process for x^*, it is clear that, given a discrepancy $x_u - x^*_u$ at the time of information arrival u, the value of the minimized cost starting at u will be identical to the value at $u + n$ (n being an integer) if the discrepancy is the same at that time. The discrepancy $x - x^*$ is therefore a sufficient state variable for the value function at times of information arrival. Since there will never be an adjustment between information arrivals, it suffices to consider the value function just at times of information arrivals.

2.2 Solving the Optimization Problem

It is never worthwhile to correct small deviations from the optimal level of the control variable because adjustment costs are lump-sum. Also, as the adjustment costs incurred depend neither on the state before adjusting nor on the size of the adjustment, the agent always adjusts to the same level of discrepancy $(x - x^*)$, which is zero in the case of a driftless process. Given the quadratic nature of the flow costs incurred by departing from the frictionless optimal value, the discrepancies which trigger an upward

[12] When there are adjustment costs, if an adjustment takes place at the time of an information arrival, then it is obvious that there will be no adjustments before the next information arrival. It is not as obvious, although it is fortunately true, that when there is no adjustment when information arrives, there will be no adjustments before the next information arrival either. Thus, the assumption of no drift simplifies the problem enormously. When there is a drift, it is necessary to determine whether to adjust and the size of adjustments at all times.

adjustment and a downward adjustment are symmetric around zero. Thus, the optimal control policy is not to adjust between information arrival times, and to reset discrepancy to zero just after information arrival if its absolute level is greater or equal to a given value, which we call s. This rule is clearly time-and-state dependent. The value of s is what remains to be determined.

We can determine it by solving a discrete time stochastic dynamic programming problem, where at each time of information arrival, the agent decides either to pay the adjustment cost and to adjust the discrepancy to zero, and consequently x to x^*; or to wait until the next period of information arrival. Without loss of generality, we set the length of the interval between information arrivals to one. Formally,

$$V(y) = \min\{B(y) + e^{-\rho}E_tV(y-\epsilon);\ k + B(0) + e^{-\rho}E_tV(-\epsilon)\} \qquad (2)$$

where the function B represents the expected discounted cost of departing from the frictionless optimal level of the control variable between now and the time of the next information arrival and ϵ is the shock to the frictionless optimal process between t and $t+1$; which is a normal variable with mean zero and variance σ^2. The expression for B is given by[13]:

$$B(y) = \frac{y^2(1-e^{-\rho})}{\rho} - \frac{\sigma^2 e^{-\rho}}{\rho} + \frac{\sigma^2(1-e^{-\rho})}{\rho^2} \qquad (3)$$

Equation (2) is valid for every y. The right-hand side can be viewed as a transformation T of the function V. The right V is found when $T(V) = V$. Because T is a contraction mapping, $T^n(V)$ tends to V as n becomes large. Therefore, V can be found by guessing an initial value for V and iterating until convergence. After the value function is found, the optimal policy can be evaluated.

[13] All mathematical derivations are included in the Appendix of the working paper version of this paper which can be downloaded from the following web site: http://www.crde.umontreal.ca.

The second argument of the min function does not depend on y. Moreover, since $B(\cdot)$ is increasing in $|y|$, and the shock ϵ has normal distribution with mean zero[14], it is intuitive that the value function itself is increasing in $|y|$. Therefore, there exists a cutoff discrepancy s such that, above it, it is optimal to adjust and, below it, inaction is optimal. The level s is the discrepancy that makes the agent indifferent between adjusting or not adjusting:

$$B(s) + e^{-\rho} E_t V(s - \epsilon) = k + B(0) + e^{-\rho} E_t V(-\epsilon)$$

When the discrepancy is zero, it is clearly optimal not to adjust, and the right hand side of the equation is equal to $V(0) + k$. As s is the point that makes the agent indifferent between adjusting or not, the left hand-side is equal to the value function evaluated at s. Thus, the condition above can be restated as a value matching condition, which is a familiar one in the problem of optimal control with full information.

$$V(s) = V(0) + k \tag{4}$$

This condition must be satisfied both at s and at $-s$, since the value function is symmetric. Then, once the value function is obtained, this equation can be used to determine s.

Note that with infrequent information, the value matching condition plays a different role than in the full information resetting problems. In the latter, the value matching condition is a condition of consistency which is always satisfied by the value function at the resetting and trigger points, even if these points are not optimal. In our problem, it is truly an optimality condition in the sense that it is only satisfied if an optimal s is chosen. In other words, if we chose a non-optimal resetting point and calculated the value function for

[14] The relevant feature of the normal distribution for this result is that its density is decreasing in $|y|$.

this resetting policy, there would be a discontinuity in the value function at such chosen resetting point.

2.3 Numerical Results

Table 1 reports the optimal rule parameters found numerically[15] for different diffusion parameters of the frictionless optimal control process. For purposes of comparison, we also report the barrier value for the optimal rule when the agent has continuous information about the optimal value of the control variable. The optimal rule in this case is also symmetric and two-sided[16], but adjustment occurs whenever the absolute value of the discrepancy is equal to the barrier.

The first pattern to notice is that the infrequent information bands are much narrower than the continuous information ones. To build one's intuition, let us note that changing the size of the band entails a trade-off. With a barrier lower than the optimal level, the costs of being away from the optimal level decrease, but the adjustment costs increase in a larger magnitude. In the infrequent information case, a reduction in the barrier level entails a much smaller increase in adjustment costs than in the continuous control case since control is only exerted at times of information arrival. So, relatively, reducing the inaction range does not substantially increase costs.

The second pattern is that the size of the bands is much less sensitive to the variability of the frictionless optimal process in the infrequent information case. When information arrives continuously, the size of the band increases with the diffusion parameter because maintaining the size constant will imply a substantial increase in adjustment costs, while not changing much the

[15] The value function is computed numerically using a piecewise linear approximation along a grid with a large number of points. We start with some initial value for this set of points and iterate until convergence. Once the value function is obtained, the s value is found by using condition (3).

[16] To compute the optimal s for the continuous information case, we use the formula in Bonomo (2000) for the two-sided optimal rule.

costs of being away from the optimal level. The unavoidable increase in costs is minimized when the size of the band becomes wider and both costs are increased. In the infrequent information case, more variability increase the probability of larger discrepancies at times of no information. So, the expected cost of being away from the optimal level increases substantially, since it is convex, even with the same barriers. On the other hand, the increase in adjustment costs is lower relative to the continuous case. Therefore, in the infrequent information case, there is less need to balance the increase in costs by enlarging the region of inaction.

A third pattern emerges from the table: the infrequent information barriers tend to respond less to the change in the diffusion parameter when this parameter is relatively large or when the adjustment costs are small. To understand this result, notice that the difference between the two cases accentuates when the adjustments in the continuous case tend to occur at intervals that are small compared to the interval between information arrivals. Adjustments tend to occur more often in the continuous case when the adjustment cost is smaller or when the diffusion parameter gets larger. Therefore, it is in those cases that the infrequent information barriers are less responsive to changes in the diffusion parameter. As illustrated in Table 1, for a high adjustment cost, the barriers become less responsive to changes in the uncertainty parameter when it gets larger. For a small adjustment cost, the optimal barriers stay practically constant for all values of the diffusion parameter.[17]

Finally, it can be seen in the table that the size of the band seems to be relatively more sensitive to the adjustment cost in the infrequent information case. The optimal band size should equalize the adjustment cost to the benefit of adjusting now rather than continuing with discrepancy s. In the infrequent information case, this benefit is less sensitive to s, since adjust-

[17] We say "practically" because when calculated with a higher level of precision, it can be noticed that the barriers move, albeit very little.

ments are only partially state-dependent. A more substantial increase in s is therefore necessary in order to make the benefit of adjusting from s now (and then following the optimal policy) equal to a higher adjustment cost level.

3 Aggregate Effects of Macroeconomic Shocks

We saw that infrequent information changes some features of the optimal rules. We will see that infrequent information changes also the pattern of response of macroeconomic variables to aggregate shocks.

We are interested in modelling the effect of an aggregate shock on the level of disequilibrium of a macroeconomic variable, as it is done in Bertola and Caballero (1990). Disequilibrium is defined as the difference between the level of an aggregate variable in a frictionless world (without adjustment costs) and its level when infrequent information and adjustment costs are present. We then compare the effects predicted by our model with the effects obtained when only one kind of imperfection prevails: a full information adjustment cost model and an infrequent information model with no adjustment costs.

3.1 Average Effect of Shocks in the Model with Infrequent Information and Adjustment Costs

In this subsection we first develop expressions for the effects of cumulative aggregate shocks of different sizes and for the average of these effects in the model with adjustment costs and infrequent information. We then compute and analyze the effects for various configurations of the uncertainty and adjustment cost parameters.

3.1.1 Analytical expression for the average effect of shocks

We define the disequilibrium level of a macroeconomic variable as the negative of the average deviation of the control variable from its frictionless optimal level, where the average is taken over all agents. The analysis can be applied to any macroeconomic variable where the frictionless optimal level can be considered as an exogenous and driftless process. For example, in a stable economy with no inflation, we can think of the individual control variable x_i as being the individual price p_i, and the frictionless optimal individual price as being the sum of the money supply and an idiosyncratic component ($x_i^* = m + e_i$). In this case, the disequilibrium level of aggregate price will be the real money supply ($y = x^* - x = m - p$). Thus, in this example, the disequilibrium level of the macroeconomic variable corresponds to the output level or some increasing function of it.[18].

Let y be the disequilibrium level of the macroeconomic variable and consider the variation of y in one period. During this period, aggregate shocks and idiosyncratic shocks to individual frictionless optimal levels of the control variable accumulate, but no adjustment is made until the information is received. Then, adjustment is made or not, depending on the level of the individual disequilibrium revealed. We explore the effect of aggregate shocks when both the information about the aggregate shock and the information about the idiosyncratic shock are released simultaneously to all agents.

The change in the average disequilibrium between adjustment periods Δy is the average, over all individual units, of the changes in the individual frictionless levels Δx_i^* less the average of the actual changes Δx_i:

$$\Delta y = \int (\Delta x_i^*) di - \int (\Delta x_i) di. \qquad (5)$$

However, since idiosyncratic shocks sum to zero, the average change in the frictionless levels is just the aggregate shock w accumulated in the period between two adjustments:

[18] This characterization of the macroeconomic variable is standard in models for money effects on output in the state-dependent rule literature. See Caplin and Spulber (1987), Caplin and Leahy (1991), Caballero and Engel (1992,1993).

$$\int (\Delta x_i^*) di = \int (w + e_i) di = w; \qquad (6)$$

where e_i is the accumulated idiosyncratic shock of agent i in one period. On the other hand, the average of the actual changes can be broken down in two parts:

$$\int (\Delta x_i) di = p_u E_u |\Delta x_i| + p_d E_d |\Delta x_i|; \qquad (7)$$

where p_u is the fraction of upward adjustments and p_d the fraction of downward adjustments, while $E_u |\Delta x_i|$ and $E_d |\Delta x_i|$ are the average size of upward and downward adjustments respectively. The agents who adjust upwards are those who realize upon receiving the information that the optimal level of their control variable exceeds the actual level by more than s. Thus, each agent adjusts its control variable by at least s. A similar reasoning applies to the fraction of agents which adjusts downwards. The overall change can therefore be written as:

$$\Delta y = w - p_u E_u |\Delta x_i| + p_d E_d |\Delta x_i|: \qquad (8)$$

To know the effect of a cumulative aggregate shock w it is therefore necessary to know both the fraction and the average size of upward and downward adjustments. These quantities depend only on the distribution of the initial individual disequilibria when there are no idiosyncratic shocks.

Consider then a world with no idiosyncratic uncertainty. Whenever there is a positive accumulated aggregate shock, $w > 0$, some units will adjust upwards if their discrepancy is lower than $-s$. This is illustrated in Figure 1. Let F_t be the cumulative distribution of the discrepancy level before the cumulative aggregate shock w. Then p_u is given by $F_t(-s + w)$[19] and p_d is zero, since no unit will increase its discrepancy. The size of the upward

[19] Let d_i be the discrepancy of an unit before receiving the shock. Thus the discrepancy after receiving a positive shock w to the frictionless optimal level is $d_i - w$. The unit will adjust if $d_i - w < -s$, which is equivalent to $d_i < -s + w$.

adjustment for a unit with disequilibrium z is always equal to the absolute value of the final disequilibrium, that is $|z - w| = -(z - w)$. The effect is totally symmetric in the case of a negative aggregate shock.

The existence of idiosyncratic shocks increases the frequency of both positive and negative adjustments at times of information arrival. Without further assumptions, we cannot tell if it will magnify or dampen the effect of aggregate shocks. If the distribution of price deviations is decreasing in the absolute size of the deviations, then the idiosyncratic uncertainty tends to attenuate the effect of aggregate shocks. This is because a positive aggregate shock, for example, tends to simultaneously make the left side of the distribution of price deviations thicker while leaving the right side empty. Then the added idiosyncratic shocks cause many more upward adjustments than downward adjustments. As a result, the effect of a positive shock is dampened. Obviously, a symmetric version of the same mechanism works for negative shocks too.

We first derive an analytical expression for the average relative effect of a cumulative aggregate shock of a given size[20]:

$$E\left[\frac{\Delta y}{w}\,|w\right] = 1 - \frac{1}{w}\left[\int_{-S}^{2}\int_{z-w+S} - \left(\frac{z-w-e_i}{\sigma_i}\right)\phi\left(\frac{e_i}{\sigma_i}\right)de_i - \int_{-\infty}^{z-w-S}\left(\frac{z-w-e_i}{\sigma_i}\right)\phi\left(\frac{e_i}{\sigma_i}\right)de_i\right]dF(z) \qquad (9)$$

where F is the ergodic (average) distribution of price deviations (see next subsection) and ϕ is the normal distribution density. Then these effects are averaged according to the likelihood of each shock size to yield an expression for the average relative effect of an unspecified aggregate shock:

[20] The derivation of the next two formulas and other technical details are included in the appendix referred to in footnote 13.

$$E\left[\frac{\Delta y}{w}\right] = 1 - \int_{-\infty} \frac{1}{w\sigma_A}\phi\left(\frac{w}{\sigma_A}\right)\left\{\int_{-s}\left[\int_{z-w+s}\left(\frac{z-w-e_i}{\sigma_i}\right)\phi\left(\frac{e_i}{\sigma_i}\right)de_i - \int_{-\infty}^{z-w-s}\left(\frac{z-w-e_i}{\sigma_i}\right)\phi\left(\frac{e_i}{\sigma_i}\right)de_i\right]dF(z)\right\}dw \qquad (10)$$

3.1.2 The ergodic distribution of deviations

Although there is no invariant distribution of deviations in the presence of aggregate shocks, we can compute the average (ergodic) distribution which coincides with the distribution that would remain invariant if all units only had idiosyncratic shocks. The ergodic distribution for this class of rules has an atom at zero, since many units adjust their discrepancy simultaneously to zero at the time of information arrival[21].

Despite the fact that aggregate shocks occur continuously and are always small in magnitude, adjustments are large, infrequent and have a large degree of simultaneity. Simultaneity results from the infrequent release of information about aggregate shocks, which makes the magnitude of news relatively large, even though innovations are small and occur continuously.

[21] The conditions that determine the ergodic distribution are given in the Appendix referred to in footnote 13. Basically, the distribution is derived by iterating on the following condition:

$$f_{t+1}(z) = \int_{-z-s}^{-z+s} f_t(z+v)\frac{1}{\sigma}\phi\left(\frac{v}{\sigma}\right)dv + \left(1 - \int_{-s}^{s} f_t(z)dz\right)\phi\left(\frac{-z}{\sigma}\right);$$

with ϕ the normal distribution density, until convergence (at the 1d-06 level).

3.1.3 Results

Figure 2 shows the ergodic distribution corresponding to two different values of the diffusion parameter. We observe that the higher the diffusion parameter, the flatter the density curve and the higher the probability associated with the atom at zero. This is consistent with the fact that a higher variance triggers more adjustments at times of information arrival, giving more weight to the atom. Additionally, as there is more movement in the deviation between adjustment times, the density gets flatter. These results tell us that the higher the aggregate uncertainty, the greater the simultaneity of actions. Intuitively, with higher uncertainty, information arrivals bring about more news. A large piece of news, that is a large cumulative aggregate shock since the last information arrival, triggers simultaneous adjustments from a large number of agents.

Since the ergodic distributions are symmetric and decreasing in the absolute size of the price deviation, we can rely on the analysis of the effect of cumulative aggregate shocks developed in section 3.1.1. Figure 3 shows that the relative effect of a shock is decreasing with the absolute size of the shock, as anticipated in the previous description about the effect of aggregate shocks. Table 2 shows the average effect of a shock for different total variances of shocks and different decompositions of this total variance between aggregate and idiosyncratic variances of shocks. The average effect decreases when we keep aggregate uncertainty constant and increase idiosyncratic uncertainty, as anticipated in section 3.1.1. When we keep idiosyncratic variance constant and increase aggregate variance, the average effect is also reduced[22]. This result follows solely from the higher likelihood of large shocks, which have a lower relative effect, since the aggregate uncertainty does not affect the impact of any specific aggregate shock. When we keep total uncertainty

[22] It should be noted that the effect is not reduced as much as one might have expected, but one has to realize that the band increases as the total uncertainty increases, thereby lowering the effect.

constant but increase the relative weight of idiosyncratic shocks, the effect is reduced indicating that the influence of idiosyncratic uncertainty on each specific shock takes pre-eminence over the way shocks are averaged. Finally, we observe that the size of adjustment costs has a very important impact on the results. This is because the size of the bands is very sensitive to adjustment costs. A reduction in adjustment costs reduces the size of the band substantially, decreasing the proportion of units that do not adjust and, as a consequence, the relative effect of an aggregate shock.

3.2 Comparison with other models

In this section we compare the features of aggregate effects in the model with infrequent information and lump-sum adjustment costs to the ones obtained in models where only one kind of imperfection is present.

3.2.1 Adjustment cost with full information

A lot of work has been done on the aggregate effects of shocks in adjustment cost models with full information, as referred to in the introduction. Caballero and Engel (1992) develop a method to quantify the average effect of aggregate shocks and apply it to assess output effects. However, their method measures instantaneous effects, and our objective is to compare aggregate effects in both infrequent information and full information models during the same time horizon[23]. The convenient time horizon we choose is the time between information arrivals.

We want to evaluate the effect generated by cumulative aggregate shocks of different sizes. First, it should be noticed that two cumulative aggregate shocks of the same size may have different effects even if the initial distribu-

[23] The relative instantaneous average effect of an aggregate shock in a full information adjustment cost model with idiosyncratic uncertainty is always one since the ergodic distribution has density zero at the trigger points. The instantaneous effect in the infrequent information model is also one.

tion of price deviations is the same. This is due to the hysteresis built in the models: the path of aggregate shocks matter, not just their cumulative sum. Given this feature, to obtain a unique measure of the relative effect of a cumulative aggregate shock of a given size, we have to average the effect of each shock of a given size with different components by the relative likelihood of its components. We therefore discretize the time and state space according to the methodology described in Bertola and Caballero (1990). We perform Monte Carlo simulations of aggregate shocks, drawing paths according to their likelihood, and calculate the relative aggregate effect for each path. We then classify the cumulative shock sizes in small intervals and average the effects of cumulative shocks in each class, in order to obtain a representative average effect for each category of shock size. We also calculate a global average of all simulations.

In Figure 4 we graph our simulations according to each class of size shock[24]. We notice that, in contrast with the infrequent information case in Figure 3, the relative effect of shocks tends to stay constant, except for shocks that are very small. Average results are shown in Table 2. Average effects tend to be of similar magnitude but do not always go in the same direction. A higher aggregate uncertainty, all other parameters being kept constant, tends to increase the average effect, contrary to what we found in the infrequent information model.

If we interpret our model in terms of the pricing application described earlier, we could think of the aggregate shock as being an aggregate demand shock. Then, our model implies that a small aggregate shock has a relatively large effect on output, while large shocks tend to have a relatively lower effect. Another implication is that in more unstable periods (subject to more aggregate shocks), the effect of a given aggregate shock tends to be reduced.

[24] The Monte Carlo simulation is based on 10000 replications. The state space is $2N + 1$, where $N = \mathrm{round}(\frac{s}{sig} \times \mathrm{sqrt}(1000))$: Tests done for a finer grid and a higher number of replications gave very similar results.

These implications constrast with those of the full information model, where the average effect of a shock does not depend on it size and shocks tend to have more effect in unstable macroeconomic environments. Both models predict that in an unstable microeconomic enviroment aggregate demand shocks have smaller effects.

3.2.2 Infrequent information with no adjustment costs

Caballero (1989) has worked out a model with no adjustment costs where part of the information relevant to each firm arrives at infrequent intervals, either through the payment of an information gathering cost or by observing the action of another firm which just paid its information gathering cost. There are some aggregate effects that come from sluggish adjustment to innovations in the frictionless optimal level of the control variable due to infrequent information. However, these effects do not last more than the time interval between information collections.

A nested simple version of our model with no adjustment cost would entail a full adjustment at every period of information arrival. Any shock will have a full effect until the time of information arrival, when the effect will be eliminated by the full adjustment of all units. Therefore, if we use the same time interval we used above to measure the effect of a cumulative aggregate shock in the models with adjustment costs, we find no aggregate effect.

4 Final Comments and Extensions

The need to put information gathering costs and adjustment costs together to yield optimal rules that are both time and state dependent has long been recognized by researchers in the macroeconomic literature. This paper makes a step forward in this direction by assuming infrequent information about the optimal control variable and lump-sum adjustment costs, and deriving resulting optimal rules that are both time and state dependent. From this point it does not seem difficult to endogenize information arrival by introducing costs of information collection. However, as argued in the introduction, infrequent exogenous information arrival is realistic per se in various contexts. A more difficult task is to generalize the current model to stochastic processes that are not martingales: then there may be adjustments between intervals of information collection, and inertia bands in this interval should depend on the remaining time before the next information arrival. Also, the same difficulties would appear if we extended the model to allow part of the stochastic component to be continuously observed.

In the aggregation of our simple rules we used a specific assumption: all agents receive information at the same time. This assumption, although extreme, captures a realistic feature of the economic world: some important information such as the release of macroeconomic statistics, tends to reach a lot of economic agents at the same time. Using this assumption, we arrived at the result that a higher aggregate uncertainty increases the simultaneity of agents' actions. Other distinctive results are that the effect of cumulative aggregate shocks tends to decrease with the absolute size of the aggregate shock, and that the larger the aggregate uncertainty, the lower the average effect of shocks. We also find implications which are shared with models of adjustment cost with full information: a higher idiosyncratic uncertainty and a lower adjustment cost both tend to reduce the effect of an aggregate shock. The aggregation part could be extended to include heterogeneity in information arrival times and information externality among agents, as in Caballero (1989). However, given the differences between information structures that are appropriate for various areas of macroeconomics where adjustment costs apply, we leave these extensions for specific applications of the model.

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Table 1
Optimal Barriers for Various Levels of Uncertainty
($\rho = 0.025$)

	σ	Infrequent Information	Continuous Information
$k = 0.01$	0.05	0.081	0.1113
	0.10	0.094	0.1570
	0.15	0.098	0.1921
	0.20	0.100	0.2217
$k = 0.001$	0.05	0.032	0.0624
	0.10	0.032	0.0881
	0.15	0.032	0.1079
	0.20	0.032	0.1245

Table 2
Average Effect of Shocks
for Various Configurations of Aggregate and Idiosyncratic Uncertainty

σ	σ_A	σ_I	k	Infrequent Information		Continuous Information	
				s	Average Effect	s	Average Effect
0.05	0.043	0.025	0.01	0.081	0.504	0.1113	0.513
0.10	0.043	0.090	0.01	0.094	0.152	0.1570	0.158
0.10	0.090	0.043	0.01	0.094	0.340	0.1570	0.399
0.05	0.025	0.043	0.01	0.081	0.383	0.1113	0.272
0.05	0.025	0.043	0.001	0.032	0.058	0.0624	0.136

Figure 1 - Effect of a Positive Aggregate Shock *w* (with no idiosyncratic uncertainty)

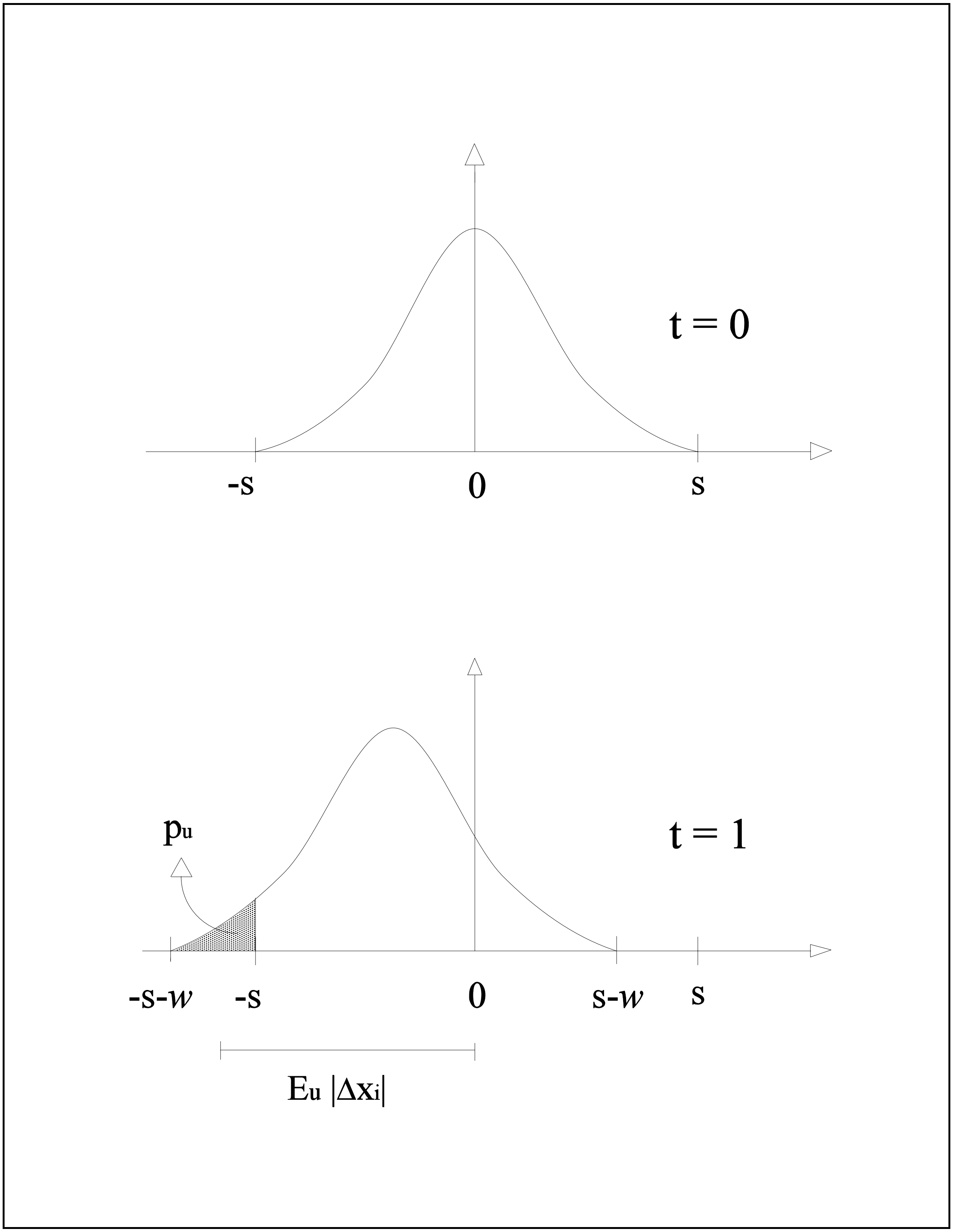


Figure 2 – Ergodic Distributions

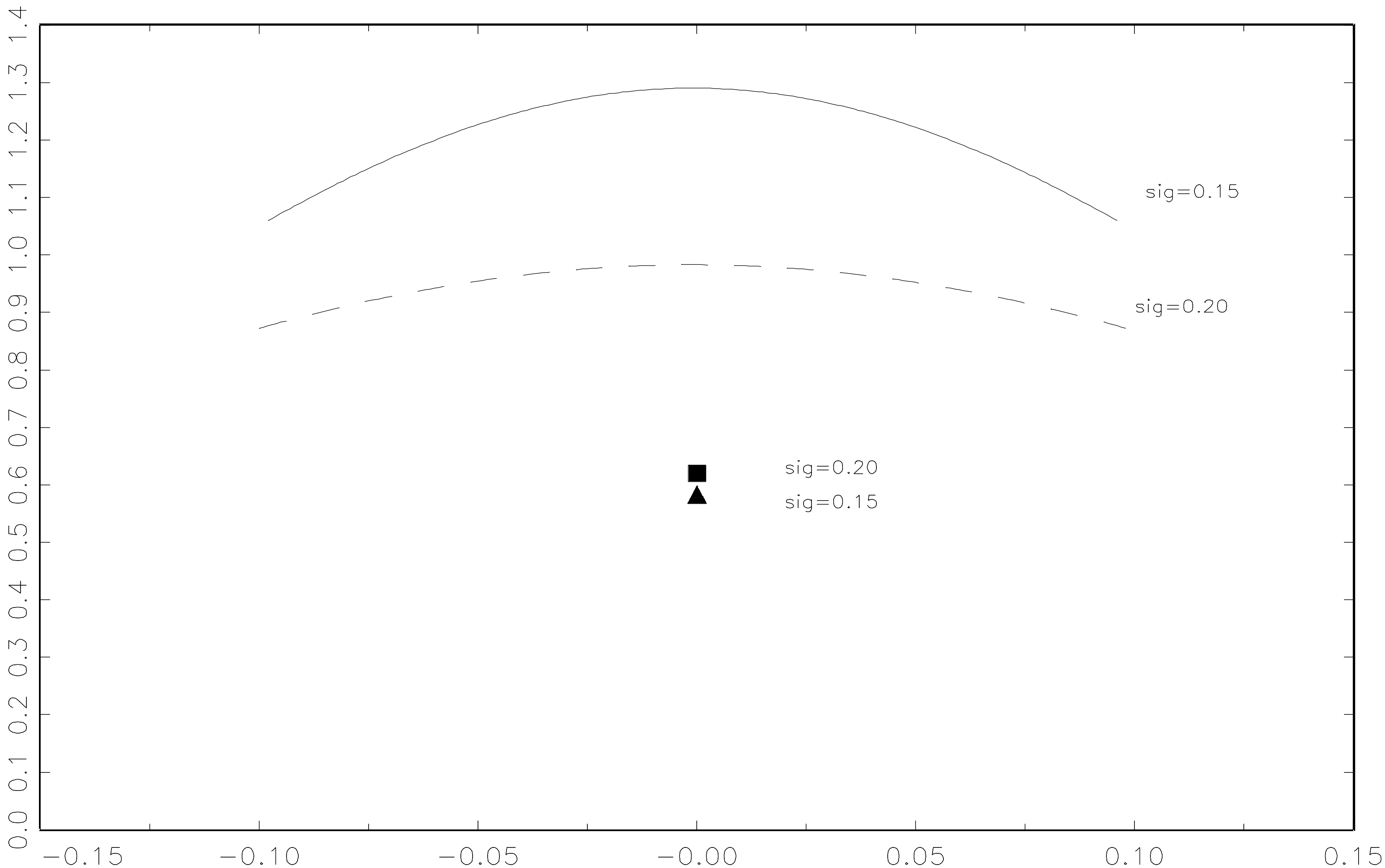


Figure 3 – Average Effect of Shocks Per Size of Shock

sig=0.050,siga=0.043,sigi=0.025,k=0.010,s=0.0810

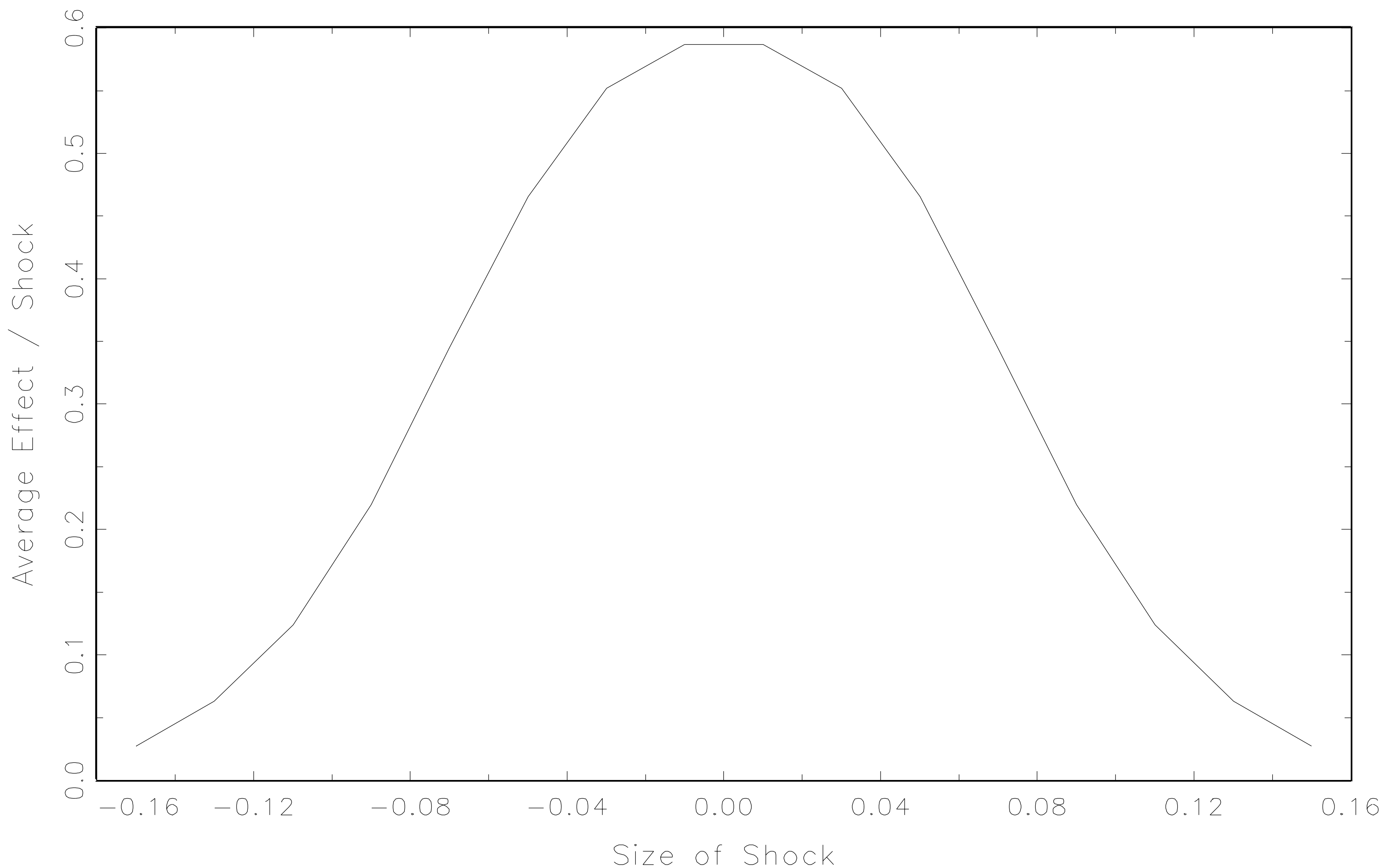
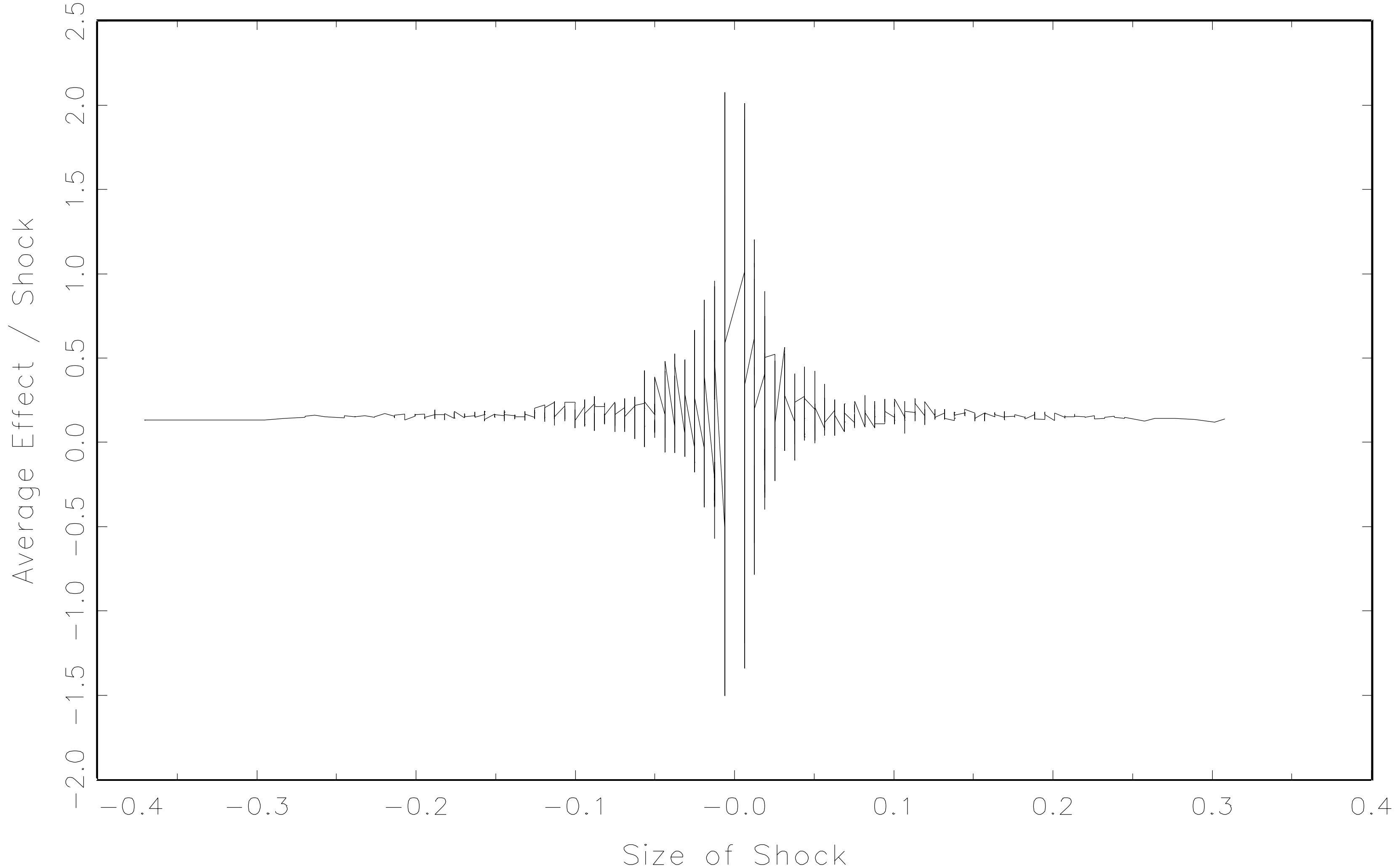


Figure 4 - Average Effect of Shocks Per Size of Shock

sig=0.100,siga=0.043,sigi=0.090,k=0.010,s=0.1570



Appendix

A: Calculating the function B

The function B represents the expected discounted cost of departing from the frictionless optimal level of the control variable between now and the time of the next information arrival. It is derived as follows

$$
\begin{aligned}
B(y) &= E_t\left(\int_0 e^{-\rho z}(x_t - x^*_{t+z})^2 dz\right) \\
&= E_t\left(\int_0 e^{-\rho z}(y + \sigma(w_{t+z} - w_t))^2 dz\right) \\
&= \int_0 e^{-\rho z} E_t(y + \sigma(w_{t+z} - w_t))^2 dz \\
&= \int_0 e^{-\rho z}(y^2 + \sigma^2 E_t(w_{t+z} - w_t)^2) dz \\
&= \int_0 e^{-\rho z} y^2 dz + \int_0 e^{-\rho z}\sigma^2 z dz \\
&= \frac{y^2(1 - e^{-\rho})}{\rho} - \frac{\sigma^2 e^{-\rho}}{\rho} + \frac{\sigma^2(1 - e^{-\rho})}{\rho^2}
\end{aligned}
$$

In the second equality, we decompose the discrepancy in $t+z$ into the sum of the discrepancy in t, which is y; and the change in x^* between t and $t+z$. The third equality results from applying Fubini's theorem, while the two next ones use respectively the conditional independence of increments of the Wiener process and the formula for their variance. The last equality is obtained by calculating the integrals, the second one with an integration by parts.

B: Derivation for the expression of the effect of a cumulative aggregate shock when there is idiosyncratic uncertainty

Our general formula (??) in the text tells us that to evaluate the effect of a cumulative aggregate shock W, it is necessary to know both the fraction and the average size of upward and downward adjustments. Both depend on the initial distribution of agents' deviations and on the cumulative idiosyncratic shock which affected the optimal level of the control variable for each agent. Since realizations of the idiosyncratic shocks across the economy are generally unknown, we evaluate the average effect of a known aggregate shock w, by averaging over all possible realizations of the e_i shocks weighted by their likelihood. As a first step, suppose that all agents have the same initial discrepancy z. Then the effect of an aggregate shock w will be:

$$E[\Delta y|w;z] = w - \left(1 - \Phi\left(\frac{z-w+s}{\sigma_i}\right)\right)\int_{z-w+s}^{\infty} -(z-w-e_i)\frac{\phi(\frac{e_i}{\sigma_i})}{\sigma_i\left(1-\Phi\left(\frac{z-w+s}{\sigma_i}\right)\right)}de_i$$
$$+\Phi\left(\frac{z-w-s}{\sigma_i}\right)\int_{-\infty}^{z-w-s}(z-w-e_i)\frac{\phi(\frac{e_i}{\sigma_i})}{\sigma_i\Phi\left(\frac{z-w-s}{\sigma_i}\right)}de_i$$

The term between parentheses multiplying the first integral is the probability that a discrepancy of level z, after accounting for the known aggregate shock w and the normally distributed idiosyncratic shocks, becomes smaller than $-s$, triggering an upward adjustment. Thus, the first integral is the expected size of the upward adjustment, conditioned on the occurence of such an adjustment. The second integral and the term that multiplies it apply to downward adjustments and have similar interpretations.

The initial discrepancies of the units at t, rather than being concentrated on a specific value of z, are distributed according to some distribution F_t. Assuming that there are many units at each position z, such that the frequency of idiosyncratic shocks for all units at a given position can be well approximated by its probability distribution, we can average the effect of an aggregate shock, as calculated above for a given z, according to the distribution F_t of the z's. Then,

$$E[\Delta y|w;F_t] = w - \int_{-s}^{s}\left[\int_{z-w+s}^{\infty} -\left(\frac{z-w-e_i}{\sigma_i}\right)\phi\left(\frac{e_i}{\sigma_i}\right)de_i - \int_{-\infty}^{z-w-s}\left(\frac{z-w-e_i}{\sigma_i}\right)\phi\left(\frac{e_i}{\sigma_i}\right)de_i\right]dF_t(z)$$

Next, we express the effect as a ratio, dividing the above expression by w, resulting in:

$$E\left[\frac{\Delta y}{w}|w;F_t\right] = 1 - \frac{1}{w}\int_{-s}^{s}\left[\int_{z-w+s}^{\infty} -\left(\frac{z-w-e_i}{\sigma_i}\right)\phi\left(\frac{e_i}{\sigma_i}\right)de_i\right.$$
$$\left.-\int_{-\infty}^{z-w-s}\left(\frac{z-w-e_i}{\sigma_i}\right)\phi\left(\frac{e_i}{\sigma_i}\right)de_i\right]dF_t(z)$$

Now we take the expectation with respect to w, yielding:

$$E\left[\frac{\Delta y}{w} \mid F_t\right] = 1 - \int_{-\infty} \frac{1}{w\,\sigma_A}\phi\left(\frac{w}{\sigma_A}\right)\left\{ \int_{-s} \left[\int_{z-w+s} -\left(\frac{z-w-e_i}{\sigma_i}\right)\phi\left(\frac{e_i}{\sigma_i}\right)de_i - \int_{-\infty}^{z-w-s} \left(\frac{z-w-e_i}{\sigma_i}\right)\phi\left(\frac{e_i}{\sigma_i}\right)de_i \right] dF_t(z) \right\} dw$$

The expression above evaluates the average effect of a shock for a given initial distribution of deviations. Finally, taking expectations with respect to the distribution of deviations (and using Fubini's theorem), we arrive at:

$$E\left[\frac{\Delta y}{w}\right] = 1 - \int_{-\infty} \frac{1}{w\,\sigma_A}\phi\left(\frac{w}{\sigma_A}\right)\left\{ \int_{-s} \left[\int_{z-w+s} -\left(\frac{z-w-e_i}{\sigma_i}\right)\phi\left(\frac{e_i}{\sigma_i}\right)de_i - \int_{-\infty}^{z-w-s} \left(\frac{z-w-e_i}{\sigma_i}\right)\phi\left(\frac{e_i}{\sigma_i}\right)de_i \right] dF(z) \right\} dw$$

where F is the ergodic (average) distribution of deviations.

C: Derivation of the conditions determining the ergodic distribution

In this appendix we derive the equations which determine the ergodic distribution.

Let $f_t(\cdot)$ be the density function for price deviations different from zero at time t, immediately after information arrival and adjustments are made. Let $P_t(0)$ be the fraction of units with price deviation zero at time t, after the adjustments are made. Let v_i be the total cumulative shock to the frictionless optimal level of the control variable of unit i during the period of time without information. So, $v_i = w + e_i$ and v_i is normally distributed with zero mean and variance $\sigma^2 = \sigma_A^2 + \sigma_I^2$.

It is clear that $f_{t+1}(z) = 0$ for $z < -s$ or $z > s$. For $-s < z < s$, and $z \neq 0$, f_{t+1} is given by:

$$f_{t+1}(z) = \int_{-z-s}^{-z+s} f_t(z+v)\frac{1}{\sigma}\phi\left(\frac{v}{\sigma}\right)dv + P_t(0)\phi\left(\frac{-z}{\sigma}\right)$$

The fraction of units at zero in $t+1$ relates to the distribution in t in the following way:

$$P_{t+1}(0) = \int_{-s}^{s} f_t(z)\left[\Phi\left(\frac{z-s}{\sigma}\right) + 1 - \Phi\left(\frac{z+s}{\sigma}\right)\right] dz + P_t(0)\phi(0)$$

For the distribution to be well defined it has to satisfy for all t:

$$P_t(0) + \int_{-s}^{s} f_t(z)dz = 1$$

Making $P_{t+1} = P_t$ and $f_{t+1} = f_t$ in the conditions above determines the ergodic distribution. Notice that any two of the three conditions above imply that the third one is satisfied. Only two conditions are therefore necessary to determine the ergodic distribution.